Pairing correlation in nuclear matter from Skyrme force

Satoshi Takahara, Naoki Onishi, and Naoki Tajima

Institute of Physics, College of Arts and Sciences,

University of Tokyo, Komaba, Meguro, Tokyo 153, Japan

February 4, 2008

Abstract

The properties of pairing correlation in nuclear matter are investigated by using various versions of Skyrme forces. Truncation of states involving pairing correlation, necessary due to zero range nature of the Skyrme force, is discussed in detail. A plateau appears in pairing gap versus cutoff for each force. We propose to choose the cutoff parameter in the middle of the plateau so that the parameterization is independent of nuclides.

The mean field theory with the Skyrme force [1, 2] has been successful in describing properties of the atomic nucleus. Bulk features such as binding energies and radii have been reproduced well and, in addition, more detailed properties such as fission barriers, giant resonances, single-particle spectra and low-lying coherent particle-hole and particle-particle excitations have been studied. Various versions of the force have been constructed for specific purposes.

Dobaczewski et al. [3] proposed the SkP force in order to describe the pairing properties. While pairing correlation contributes to various properties of nuclei, one of the most interesting topics is the formation of skins and halos in neutron excess nuclei. In these nuclei the neutron Fermi energy rises close to but below zero so that unoccupied neutron orbitals just above the Fermi level are unbound. Therefore the pairing interaction scatters particle pairs from bound states to continuum states. This process can have sizable amplitude in the peripheral region and the formation of neutron skin or halo can be enhanced. Pairing correlation in neutron excess nuclei is also discussed in other articles, e.g. ref. [4].

1

In the Hartree-Fock (HF)+BCS method, the mean field is solved self-consistently but the pairing field is separately treated. The pairing interaction is usually taken to be so called seniority force or the monopole force. The force is independent of the orbital so that gap equations are decoupled from the HF field and the whole scheme becomes extremely simple. For each nucleus, the strength of matrix element G is determined by, e.g. fitting to experimental odd-even mass staggering. The matrix element G must, however, be dependent on quantities other than N and Z, e.g., the density and deformation. In order to deal with the pairing gap more properly, it is desirable to utilize interactions independent of nuclides like the Skyrme force as a more fundamental approach.

The treatment of HF+BCS including continuum states may suffer from unlocalized density due to unphysical "particle gas" surrounding the nucleus. As shown by Dobaczewski et al. [3], Hartree-Fock-Bogoliubov (HFB) theory is free from this difficulty. In this theory both the mean field and the pairing field are treated on an equal footing and determined simultaneously from the same interaction.

Though HFB has been performed with finite range Gogny force [5], it is practical to employ HFB with the Skyrme force in describing neutron excess nuclei. In using Gogny force the expansion in terms of the harmonic oscillator basis provides a poor approximation to continuum states, even when a large number of oscillator basis is taken into account. Thus coordinate-space treatment is necessary for neutron-excess nuclei. The Skyrme force has the possibility of working in coordinate space. This is the reason why the Skyrme force is employed as a pairing force in describing nuclei far from stability [3].

The consistent usage of the Skyrme force is also preferable in the application of Generator Coordinate Method [6].

In the present work we investigate the properties of pairing correlation of Skyrme forces. Firstly, the formalism and notation are presented. Secondly, the strength of pairing correlation in nuclear matter is calculated by making use of various versions of Skyrme forces. The main subject of this paper is the dependence of the pairing gap on the cutoff parameter, which is necessary for zero range forces.

The Skyrme force is usually parameterized as

$$V(1,2) = t_0(1 + x_0 P_\sigma)\delta + \tfrac{1}{2}t_1(1 + x_1 P_\sigma)(\boldsymbol{k'}^2\delta + \delta\boldsymbol{k}^2) + t_2(1 + x_2 P_\sigma)\boldsymbol{k'} \cdot \delta\boldsymbol{k}$$
$$+\tfrac{1}{6}t_3\rho^\alpha(1 + x_3 P_\sigma)\delta + iW_0(\boldsymbol{\sigma_1} + \boldsymbol{\sigma_2}) \cdot \boldsymbol{k'} \times \delta\boldsymbol{k}.$$

$$(1)$$

Following the notation employed in ref. [3], particle and pairing density matrices ρ and $\tilde{\rho}$ are defined as

$$\rho(\boldsymbol{r}\sigma q; \boldsymbol{r}'\sigma'q') = \langle \Phi \mid a^\dagger_{\boldsymbol{r}'\sigma'q'} a_{\boldsymbol{r}\sigma q} \mid \Phi \rangle, \tag{2}$$

$$\tilde{\rho}(\boldsymbol{r}\sigma q; \boldsymbol{r}'\sigma'q') = (-2\sigma')\langle \Phi \mid a^\dagger_{\boldsymbol{r}'-\sigma'q'} a^\dagger_{\boldsymbol{r}\sigma q} \mid \Phi \rangle. \tag{3}$$

Owing to the zero range nature of the Skyrme force, the energy functional depends on only the local properties of the density matrices. In HFB theory the energy density $\mathcal{H}(\boldsymbol{r})$ can be expressed as a function of the particle density $\rho(\boldsymbol{r})$, the kinetic energy density $\tau(\boldsymbol{r})$, the pairing density $\tilde{\rho}(\boldsymbol{r})$ and the pairing kinetic-energy density $\tilde{\tau}(\boldsymbol{r})$. These are defined as

$$\rho(\boldsymbol{r}) = \sum_{\sigma q} \rho(\boldsymbol{r}\sigma q; \boldsymbol{r}\sigma q), \quad \tau(\boldsymbol{r}) = \boldsymbol{\nabla}' \cdot \boldsymbol{\nabla} \sum_{\sigma q} \rho(\boldsymbol{r}\sigma q; \boldsymbol{r}'\sigma q) \mid_{\boldsymbol{r}'=\boldsymbol{r}}, \tag{4}$$

$$\tilde{\rho}(\boldsymbol{r}) = \sum_{\sigma q} \tilde{\rho}(\boldsymbol{r}\sigma q; \boldsymbol{r}\sigma q), \quad \tilde{\tau}(\boldsymbol{r}) = \boldsymbol{\nabla}' \cdot \boldsymbol{\nabla} \sum_{\sigma q} \tilde{\rho}(\boldsymbol{r}\sigma q; \boldsymbol{r}'\sigma q) \mid_{\boldsymbol{r}'=\boldsymbol{r}}. \tag{5}$$

In nuclear matter of isobaric symmetry, Hamiltonian density is given by

$$\mathcal{H} = \frac{\hbar^2}{2m}\tau + (\frac{3}{8}t_0 + \frac{1}{16}t_3\rho^\alpha)\rho^2 + \frac{1}{16}\left\{3t_1 + t_2(5 + 4x_2)\right\}\rho\tau$$
$$+ \left\{\frac{1}{8}t_0(1 - x_0) + \frac{1}{48}t_3(1 - x_3)\rho^\alpha\right\}\tilde{\rho}^2 + \frac{1}{8}t_1(1 - x_1)\tilde{\rho}\tilde{\tau}.$$

$$(6)$$

The trial variational function is taken to be of the BCS type,

$$\mid \Phi \rangle = \prod_{q,\boldsymbol{k}} \left\{ u(k) + v(k)a^\dagger_{\boldsymbol{k}\uparrow q} a^\dagger_{-\boldsymbol{k}\downarrow q} \right\} \mid 0 \rangle, \tag{7}$$

where $\boldsymbol{k}$ is wave number vector and $k = \mid \boldsymbol{k} \mid$. Then the densities are given by

$$\rho = \frac{2}{\pi^2}\int_{k_{\min}}^{k_{\max}} k^2 v^2(k)dk + \frac{2}{3\pi^2}k_{\min}^3, \quad \tilde{\rho} = -\frac{2}{\pi^2}\int_{k_{\min}}^{k_{\max}} k^2 u(k)v(k)dk, \tag{8}$$

$$\tau = \frac{2}{\pi^2}\int_{k_{\min}}^{k_{\max}} k^4 v^2(k)dk + \frac{2}{5\pi^2}k_{\min}^5, \quad \tilde{\tau} = -\frac{2}{\pi^2}\int_{k_{\min}}^{k_{\max}} k^4 u(k)v(k)dk. \tag{9}$$

In the above equations $k_{\min}$ and $k_{\max}$ are defined in terms of a cutoff energy E_c as

$$k_{\min}^2 = k_F^2 - \frac{2m^*(\rho)}{\hbar^2}E_c, \quad k_{\max}^2 = k_F^2 + \frac{2m^*(\rho)}{\hbar^2}E_c \tag{10}$$

where $m^*(\rho)$ is the effective mass of nucleon and $k_{\rm F}$ is the Fermi momentum of normal state. In order to express the pairing gap as a function of density, a constraint $\rho = \rho_0$ is imposed in minimizing the Hamiltonian density (6). For this purpose, we utilize the following Routhian,

$$\mathcal{H}' = \mathcal{H} - \lambda\rho + \frac{c}{2}(\rho - \rho_0)^2. \tag{11}$$

In the r.h.s., λ is the Lagrange multiplier and the third term having a positive constant c is included to stabilize the convergence procedure at negative curvature points. If we put $u(k) = \cos\theta(k)$ and $v(k) = \sin\theta(k)$, variation with respect to $\theta(k)$ yields an equation,

$$\tan 2\theta(k) = -\frac{\dfrac{\partial\mathcal{H}}{\partial\tilde{\rho}} + \dfrac{\partial\mathcal{H}}{\partial\tilde{\tau}}k^2}{\dfrac{\partial\mathcal{H}}{\partial\rho} + \dfrac{\partial\mathcal{H}}{\partial\tau}k^2 - \lambda + c(\rho - \rho_0)}. \tag{12}$$

In solving eqs. (8), (9) and (12), a naïve iteration procedure does not work when the pairing is weak and special manipulations are necessary to obtain the solution. We confirmed that the solution actually gives a minimum by giving the Routhian small variations in $\theta(k)$ and calculating the Routhian.

The pairing gap depending on k is defined as

$$\Delta(k) = -\sum_{\boldsymbol{k}'} V_{\rm pp}(|\,\boldsymbol{k} - \boldsymbol{k}'\,|)u(k')v(k'). \tag{13}$$

The pair-scattering matrix element $V_{\rm pp}(q)$ is of the form,

$$V_{\rm pp}(q) = t_0(1 - x_0) + \frac{1}{6}t_3(1 - x_3)\rho^\alpha + \frac{1}{2}t_1(1 - x_1)q^2, \tag{14}$$

where a term depending on the angles is omitted because it vanishes through integration for the angles.

Figure 1 shows the pairing gap $\Delta(k = k_{\rm F})$ as a function of the density characterized with the Fermi momentum $k_{\rm F}$ when $E_{\rm c}$=10MeV. This value of $E_{\rm c}$ is chosen so as to correspond to widely used $2\hbar\omega$ configuration space. The results for five parameter sets of Skyrme forces (SIII[7], SkM*,[8] SGII[9], SkP[3] and SkSC4[10]) are compared. The term proportional to t_3 in eq. (1) is treated as a density dependent force. In SIII this term can be dealt with

as a three-body force. In that case, however, pairing correlation is considerably reduced: the maximum pairing gap decreases by a factor of 1/6 and only the normal state solution is obtained for density corresponding to $k_F \geq 0.7\text{fm}^{-1}$. For comparison the result for Gogny D1[5] force, a finite range force which reproduces well the pairing properties of nuclei, is taken from ref. [11]. The figure shows: (i) the gap parameter gives the maximum at $k_F \sim 0.8$ fm^{-1}, which is equivalent to about one fifth of the saturation density ($k_F \sim 1.33$ fm^{-1}), (ii) at the saturation density, with Gogny force the pairing gap of ~ 0.6 MeV still remains while with Skyrme forces only the normal state solution is obtained.

One should notice that, as a result of the zero range nature of the Skyrme force, pairing gap may diverge if the summation in eq. (13) is carried out toward infinity. This implies the necessity of a truncation of particle states for the summation. Thus we restricted the range of integration in eqs. (8) and (9). Pairing properties of nuclear matter have been investigated so far in the context of the neutron star. Jiang and Kuo [12] studied pairing in nuclear matter by making use of Skyrme forces, but they used the weak coupling approach in which pairing gap is constant and not zero only in the vicinity of the Fermi surface. The effect of cutoff on the pairing gap has not been studied yet and we extensively discuss it in the remainder of this paper.

Figure 2 shows the pairing gap for $k_F = 0.8\text{fm}^{-1}$ as a function of the cutoff parameter E_c. The pairing gap depends strongly on the cutoff around E_c=10MeV which is employed in the calculation shown in Fig. 1. At larger cutoff, however, there is a plateau for each force. The position of the plateau alters from one parameter to another and seems to be associated with the momentum transfer q at which the pair-scattering matrix element $V_{pp}(q)$ changes the sign. Beyond the plateau the gap starts to increase again.

We should notice that at high momentum transfer the pair-scattering matrix element becomes positive and hence increasing cutoff leads to including a large amount of repulsive components in the pairing interaction. Thus increasing cutoff would not be expected to increase pairing gap. Nevertheless the calculations show that the pairing gap keeps increasing together with the cutoff. It may seem strange at first sight but can be understood as follows.

Firstly let us explain the mechanism in which increasing cutoff gives rise to an unlimitedly increasing gap. The uv-factors characterizing the BCS solution are parameterized with $\theta(k)$. For large cutoff $k_{\max}$, $\theta(k)$ takes on negative values for k larger than a certain value (see eq.(12)). In consequence, pairing kinetic-energy density $\tilde{\tau}$ changes the sign whereas pairing density $\tilde{\rho}$ remains negative, because contribution from high momentum in $\tilde{\tau}$ is larger than that in $\tilde{\rho}$. Thus the term proportional to $\tilde{\rho}\tilde{\tau}$ in the energy functional (6) comes to have the effect of lowering energy, notwithstanding the fact that this term usually plays the role of raising energy.

Secondly we consider the physical meaning in such solutions. A similar circumstance, in which a repulsive interaction contributes to the pairing correlation, is reported [13] in the Relativistic Hartree-Fock-Bogoliubov (RHFB) theory in which the force parameters are determined to simulate the Gogny force. They stated that this can happen only in cases where the gap parameter as well as the interaction changes the sign as a function of the momentum. In the case of the Skyrme force the gap parameter actually changes the sign at high momentum and contributes coherently to pairing correlation. But $V_{\mathrm{pp}}(q)$ increases rapidly as a function of the momentum whereas in the case of finite range force $V_{\mathrm{pp}}(q)$ diminishes. Therefore the contribution of repulsive part of the force is fictitious in the case of the Skyrme force although it is probably realistic for RHFB. The former force is constructed so as to describe the ground state properties of nuclei, and corresponds to the lowest order expansion in momentum. Its behavior at large momentum transfer does not make sense.

If one takes the cutoff in the middle of the plateau, the dependence of the gap on the cutoff can be made reduced considerably. Thus the parameterization (including cutoff) is expected to be independent of nuclides. However, since the plateau extends over up to 150 MeV the question arises as to how to determine the cutoff parameter. Here we propose to determine cutoff such that $\Delta(k_{\max}) = 0$.

We also study the dependence of thus-defined cutoff momentum on the density. SIII and SkSC4 preserve almost constant value over the whole density whereas the others, especially

SkP, possess notable dependence. In the application of HFB to finite nuclei, the pairing gap contributes most at the nuclear surface. Therefore we decided to take the cutoff momentum to be that at $k_F \sim 0.8$ fm^{-1} where the pairing gap has maximum in Fig. 1. Table 1 shows the cutoff momentum and corresponding cutoff energy related through eq. (10).

Figure 3 shows the pairing gap $\Delta(k = k_F)$ calculated with cutoff given in table 1 as a function of the density for k_F=0.8fm^{-1}. Some features are different from Fig. 1. The peak is about three times as high as that in fig. 1 for SkP, SGII and SkM*, whereas that for SIII and SkSC4 remains small. The order of the size of gap changes. The position of the peak moves to the lower k_F (0.6 fm $^{-1}$) for SGII, SkM* and SkP.

In conclusion, we have investigated the pairing properties of the Skyrme force in nuclear matter on the basis of HFB. In particular, the dependence of pairing gap on the cutoff is studied. We have observed the appearance of a plateau in the pairing gap at a certain cutoff characteristic of each force. The formation of the plateau is attributed to the fact that the pair-scattering matrix element changes the sign at a certain transferred momentum. We propose to choose the cutoff parameter in the middle of the plateau, so that the pairing gap can be made insensitive to the cutoff parameter and the parameterization is more likely to be independent of nuclides.

References

[1] T. H. R. Skyrme, Phil. Mag. **1** (1956) 1043.

[2] D. Vautherin and D. M. Brink, Phys. Rev. **C5** (1972) 626.

[3] J. Dobaczewski, H. Flocard and J. Treiner, Nucl. Phys. **A422** (1984) 103.

[4] G. F. Bertsch and H. Esbensen, Ann. Phys. (N.Y.) **209** (1991) 327.

[5] J. Dechargé and D. Gogny, Phys. Rev. **C21** (1980) 1568.

[6] N. Tajima, H. Flocard, P. Bonche, J. Dobaczewski and P.-H. Heenen, Nucl. Phys. **A542** (1992) 355.

[7] M. Beiner, H. Flocard, Nguyen Van Giai and P. Quentin, Nucl. Phys. **A238** (1975) 29.

[8] J. Bartel, P. Quentin, M. Brack, C. Guet and H. -B. Hakansson, Nucl. Phys. **A386** (1982) 79.

[9] N. V. Giai and H. Sagawa, Phys. Lett. **106B** (1981) 379.

[10] Y. Aboussir, J. M. Dearson, A. K. Dutta and F. Tondeur, Nucl. Phys. **A549** (1992) 155.

[11] H. Kucharek, P. Ring, P. Schuck, R. Bengtsson and M. Girod, Phys. Lett. **B216** (1989) 249.

[12] M. F. Jiang and T. T. S. Kuo, Nucl. Phys. **A481** (1988) 294.

[13] H. Kucharek and P. Ring, Z. Phys. **A339** (1991) 23.

TABLE

TABLE 1. Cutoff momentum $k_{\max}$ defined such that $\Delta(k_{\max}) = 0$ and cutoff energy $E_{\rm c}$ related with $k_{\max}$ through eq. (10) for $k_{\rm F}=0.8{\rm fm}^{-1}$.

	$k_{\max}[{\rm fm}^{-1}]$	$E_{\rm c}[{\rm MeV}]$
SIII	1.547	39.05
SkM*	1.857	61.64
SGII	2.076	80.64
SkP	2.966	169.15
SkSC4	1.258	19.54

FIGURE CAPTIONS

Fig. 1. Density dependence of the pairing gap for Skyrme forces and Gogny D1 force. Here the cutoff energy is fixed as $E_c = 10\text{MeV}$.

Fig. 2. The dependence of the pairing gap on the cutoff energy for various versions of Skyrme forces. Diamonds indicate the cutoff energy determined such that $\Delta(k_{\max}) = 0$.

Fig. 3. Same as fig. 1 but with the cutoff momentum given in table 1.